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Computation of effective behavior of isotropic transverse composite in nonlinear problems

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ABSTRACT

Computational homogenization is demonstrated as a potential analysis tool that can be used directly to predict the property structure relationships of many existing classes of composites. Based on finite element simulations, using the Zebulon structures calculation code, the present study is devoted to computation and estimation the effective behavior of the nonlinear composite materials with randomly identical elastic perfectly plastic parallel cylindrical fibers distribution in linear elastic matrix phase. The parameters of effective behavior of the composite will be expressed. These parameters are the effective elastic tangent modulus, effective plastic tangent modulus and effective yield stress. The proposed method can be readily applied to compute the parameters of effective behavior composite, using a new proposed analytical expression based on the volume fraction and effective Young's modulus of each component. A good agreement between expression and simulation results is observed. Several examples are shown.

1. Introduction

One important goal of the mechanics and physics of heterogeneous materials is to derive their effective properties from the knowledge of the constitutive laws and spatial distribution of their components. Homogenization methods have been designed for this purpose. They have reached a high level of sophistication and efficiency, especially in the case of linear properties such as thermal conductivity or elasticity. They can be found in reference extended papers and textbooks like Willis (1981), Ponte-Castañeda and Suquet (1987), Sanchez-Palencia and Zaoui (1987) and Nemat-Nasser and Hori (1993) or, more recently, Suquet (1997), Besson et al. (2001), Bornert et al. (2001) and Jeulin and Ostoja-Starzewski (2001), where extensions to nonlinear properties are also proposed.

The prediction of the macroscopic stress–strain response of composite materials is related to the description of their complex microstructural behavior exemplified by the interaction between the constituents. In linear elasticity, numerous homogenization schemes have been developed such as those of Voigt-Reuss, see Mori and Tanaka (1973) and Swan and Kosaka (1997), self-consistent and the interpolative double-inclusion model, see Nemat-Nasser and Hori (1999). Extensions to multi-phase composites and linear thermo-elasticity have followed, see Camacho et al. (1990), Lielens (1999) and Pierard (2004). These relations can be homogenized at each time step according to the classical models valid in linear elasticity. However, macroscopic predictions seem to be too stiff, see Gilormini (1995).

The extension of the homogenization models to the plastic regime was pioneered by Hill (1965) and his work was followed by many others, see Ponte-Castañeda and Suquet (1998) and Chaboche et al. (2005). Basically, two main approaches were adopted. In tangent formulations, each phase follows the incremental theory of plasticity, and the composite behavior is obtained incrementally by integrating along the loading path the composite stiffness tensor obtained from the tangent stiffness tensors of each phase, using one of the available linear approximations, see Hutchinson (1970), Pettermann et al. (1999) and Ju and Sun (2001). Secant formulations deal with the plastic deformation within the context of nonlinear elasticity, and the relationship between stress and strain in each phase is given by a secant stiffness tensor, which depends on the *von Misses* equivalent stress. The overall composite stiffness tensor is computed from those of each phase through the chosen linear approximation, see Berveiller and Zaoui (1979) and Tandon and Weng (1988).

The effective elastic properties are calculated using expressions for two phase composites proposed by Hashin (1979) and Christensen (1979). Dasgupta and Bhandarkar (1992) discussed

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a method to obtain the transversely isotropic effective thermo mechanical properties of unidirectional composites reinforced with coated cylindrical fibers. An elastic contact model is developed by Hui-Zu and Tsu-Wei (1995), to predict the transverse Young's modulus, Poisson's ratio and shear modulus of unidirectional fiber composites with interfacial debonding.

Analytic expressions for elastic effective coefficients of fibrous composites with isotropic elastic constituents for square and hexagonal cells under perfect contact conditions at the interfaces are calculated in many previous works, see Molkov and Pobedria (1985), Guinovart-Díaz et al. (2001) and Rodríguez-Ramos et al. (2001). The analytical expressions for the effective properties are obtained using *Asymptotic Homogenization Method*, AHM.

Homogenized coefficients of the microstructure have studied by Moravec and Roman (2009). They have developed a numerical procedure for computing the homogenized coefficients of elastic fibrous tissue. Tuncer (2005) presents numerical calculations of the elastic properties of two cellular structures which resemble cellular polymers used in electrical applications. A power law expression with a quadratic as the exponent term is proposed for the effective Young's modulus of the systems as a function of the solid volume fraction.

Recently, in Guinovart-Díaz et al. (2011), the effective elastic moduli of two-phase fibrous periodic composites are obtained for different types of parallelogram cells based on the AHM and making use of potential methods of a complex variable and properties of elliptic and related functions. The constituents exhibit transversely isotropic properties. Raimondo et al. (2012) proposed method can be applied for predicting the elastic and failure properties of carbon fibber polymer composites for generic 3D quasi-static and high-rate loading conditions. Rodríguez-Ramos et al. (2012) presents algorithms for predicting the full set of effective coefficients for composites based on unit cells models.

More recently, in Guinovart-Díaz et al. (2013), an analytical expression for effective elastic stiffness of a fiber reinforced composite with imperfect contact between matrix and fibers are obtained using asymptotic homogenization method for parallelogram like arrangement of fibers.

The subject treated in this paper is very special and rarely studied in the literature. The reason is that it is almost impossible to find fibers with perfect plasticity in practice. However, homogenization technique, based on the notion of the *Representative Volume Element*, RVE, is used to predict the effective properties of microstructures and developing a macroscopic mechanical constitutive model. This one is related to the behavior of the two-phase. The result in the elastic–plastic regime, reaching an *exact* solution, to a few percent, of the tension test of a composite made up of random dispersion of elastic perfectly plastic parallel identical cylindrical fibers.

2. Materials and microstructures

2.1. Morphology and meshes

The chosen microstructure to simulate the mechanical behavior was a parallelepiped composite isotropic transverse in plane (xy), which contained a random dispersion of non-overlapping identical parallel cylindrical fibers, distributed in the matrix phase as shown in Fig. 1.

The so-called multiphase-element technique is used to superimpose a finite element mesh on the images of the microstructures, see Barbe et al. (2001) and Zohdi (2001). The reason is the ability of multiphase element to assign different behaviors at different integration points in the same element, depending on its location in the composite. For each integration point of a regular finite element mesh, having the size of the considered sample, the closest pixel in the image is determined and the corresponding material property is attributed to it. The previous references show that this simple meshing strategy leads to correct evaluations of mean stresses and strains inside the phases when compared to proper meshing of interfaces by nodes. On the other hand, a relatively large number of elements is required to obtain the convergence of local results close to interfaces. Bounds for the errors introduced by the use of multiphase elements and improvement methods of the quality of the integration can be found in Zohdi (2003).

2.2. Constitutive equations of the constituents

The microstructure of the considered material is supposed to be composed of an isotropic and linear elastic matrix containing an elastic-perfectly plastic fibers, with the same Young's modulus E_m, E_f respectively of matrix and fibers. The yield stress of the elastic-perfectly plastic fibers is noted R_0^f.

The average total stress and strain have been recovered under the monotone uniaxial tension, in the composite, at each load increment. Therefore, the law of effective macroscopic behavior of the composite can be presented and called *Average stress–strain curves*, see Fig. 1.

The macroscopic behavior law of the composite is obtained numerically by the curve of the average stress tension $<\sigma_{xx}>$ versus the average strain tension $<\varepsilon_{xx}>$. The numerical simulations show that $<\sigma_{xx}> = E^{eff} <\varepsilon_{xx}>$ for $(<\sigma_{xx}>) \leq R_0^f$ and $<\sigma_{xx}> = R_0^f + H^{eff} <\varepsilon_{xx}^p>$ for $(<\sigma_{xx}>) \geq R_0^f$, where $<\varepsilon_{xx}^p>$ is the average plastic strain tension in the composite. That means that the macroscopic effective behavior of the composite is elastic plastic with linear hardening. We try now to give relations of the different constants of the effective behavior: the effective Young's modulus E^{eff} and the effective tangent modulus of linear hardening H^{eff}, as functions of E_m, E_f, R_0^f and the fibbers volume fraction. The macroscopic effective behavior in the transverse plane of this composite is obtained numerically as shown in Fig. 1.

The aim of the present work is to estimate the effective behavior parameters of the composite. Starting from the behavior lows of the two phases, the parameters of effective behavior of the composite will be expressed. These parameters are the effective elastic tangent modulus $E^{eff} = tg(\alpha)$, the effective plastic tangent modulus $H^{eff} = tg(\beta)$ and the effective yield stress R_0^{eff}, as shown in Figs. 1(e) and 2. Finally, an analytical formula will be proposed based on numerical simulation to describe the effective behavior.

3. Numerical homogenization technique

Finite Element Method, FEM, is chosen to calculate the composites given in Fig. 1, using the methodology explained in Kanit et al. (2003) by Zebulon software code. In the first part of the study, the composite chosen has the following properties: $E_m = E_f =$ 80,000 MPa, $R_0^f = 50$ MPa and has been investigated under monotone uniaxial tension. The response of the composite is obtained numerically as shown in Fig. 2. The overall axial stress is plotted as a function of the overall axial strain.

The curve in Fig. 2 can be schematically decomposed into too different regimes. The first one corresponds to the initial response of the composite which is purely elastic. In this regime the only potential which matters is the free energy and the effective response of the composite can be obtained by homogenization of the only elastic phase. A good match in this first regime indicates that the elastic behavior of the composite are accurately predicted, where the effective Young's modulus of composite similar to the elastic modulus in the two phases and can be computed as: $E^{eff} = E_f = E_m$. In the second part of the curve, corresponding to relatively large

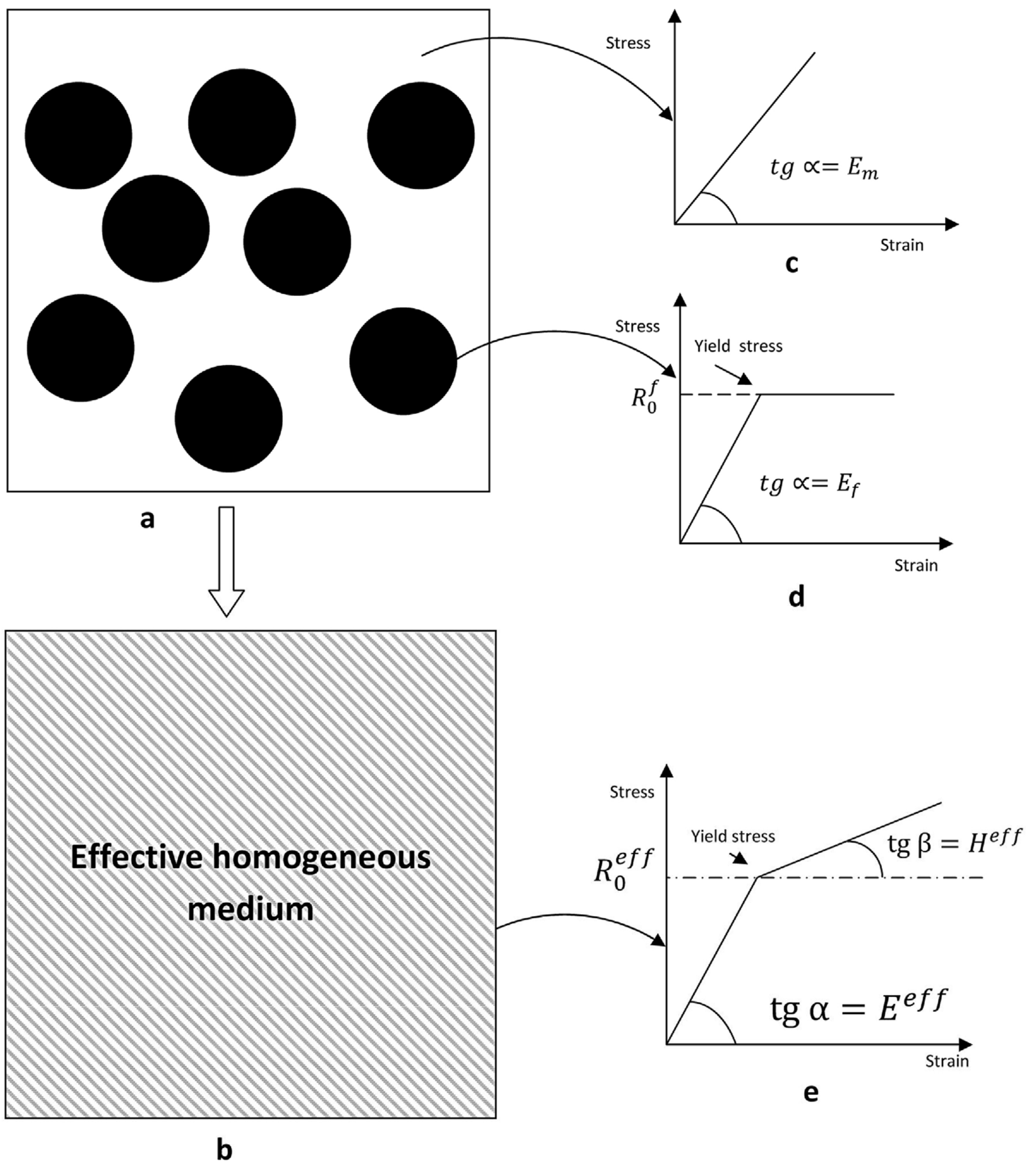


Fig. 1. Schematic description of the studied composite in transverse plane, (a) heterogeneous microstructure, (b) effective homogeneous medium, (c) behavior low of the matrix, (d) behavior low of the fibers and (e) macroscopic behavior of the effective homogeneous medium.

deformations, the behavior lows of the composite present a plastic zone with linear hardening. The obtained effective yield stress is equal to the perfectly plastic fibers one as: $R^{eff} = R_0^f$.

3.1. Representative volume element size effect

The homogenization process is done under a representative volume element, RVE. The set obtained results were sensitive to the number of fibers with keeping the fixed fibers volume fraction P. Therefore, the effect of the number of fibers on the results was checked by comparing the behavior of the microstructure considered, in the second part of the curve which present plastic zone with linear hardening, for different number of fibers $N = 1$; 50; 200 and 300, see Fig. 3. It appears that the convergence is obtained for at least $N = 300$ fibers.

Our principal interest is the determination of the dependence behavior of composite with linear hardening which is the effective plastic tangent modulus H^{eff} over the parameters: Young's modulus of the two phases, the yield stress of fibers phase and the third important parameter is the volume fraction and the correlation between this H^{eff} and the composite effective elastic properties. This approach based on numerical simulations and various values of the two phases behaviors parameters will allow to predict the effective plastic tangent modulus H^{eff} using an analytical expressions.

4. Results and discussion

4.1. Influence of yield stress

Fig. 4 shows the stress–strain curve for composites under monotone uniaxial tension in transversal plane, with different yield stress values and constant Young's modulus (80,000 MPa) and volume fraction $P = 0.3$. The influence of yield stress on effective material behavior is studied by investigation two different yield stress of the fibers phase.

It appears that the obtained effective plastic tangent modulus H^{eff} is similar in the two investigated different yield stress. This result gives only an information about the effective yield stress of the composite which is the same of the fibers one and it can be

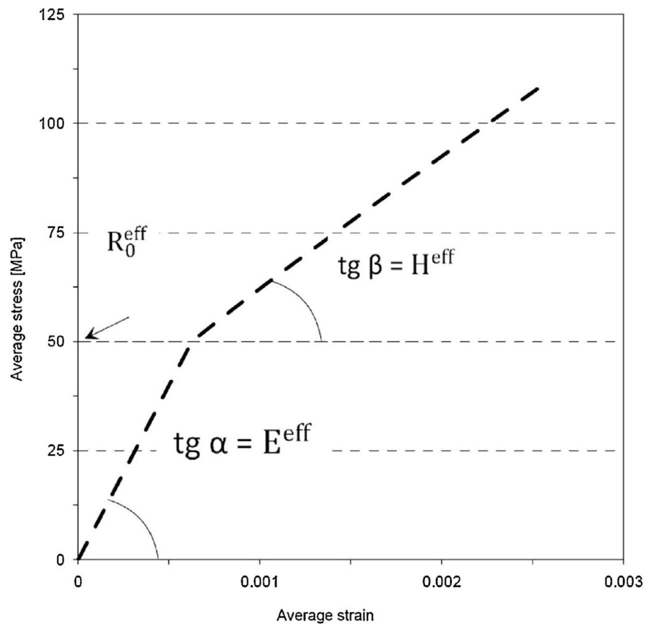


Fig. 2. Average stress–strain curves under monotone uniaxial tension in transversal plane of the composite consisting of two phases with equal elastic modulus, reinforced with 30% of cylindrical fibers volume fraction. Number of fibers $N = 300$.

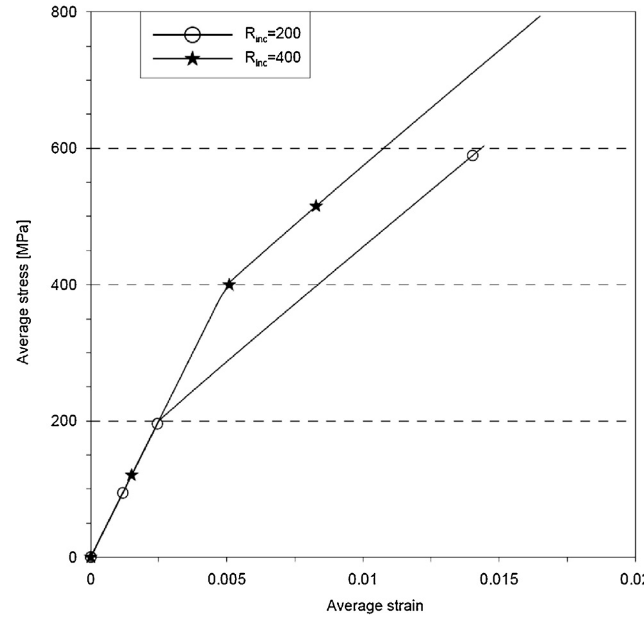


Fig. 4. Response of the transversal plane of the composite: influence of yield stress on the second part of the curve. The effective plastic tangent modulus as function of the yield stress. Two different yield stress of the fibers phase investigated 200 and 400 MPa.

seen that this yield stress variation has no effect on the effective plastic tangent modulus H^{eff}. As a consequence of this results, the potential will be given as a function of two parameters: Young's modulus and volume fraction.

4.2. Influence of fibers volume fraction

In this part, considering a same Young's modulus of the two phases $E_m = E_f = 80{,}000$ MPa. The influence of volume fraction has been investigated, by increasing this one in the fibers phase composites from 0 to 0.6, representing the percolation threshold beyond which the arrangement of the fibers becomes uniform. The random cylindrical fibers were generated and meshed by the procedure indicated in Section 2. The tensile stress–strain curves are plotted in Fig. 5.

The obtained results show that when the fibers volume fraction increases the effective plastic tangent modulus H^{eff} decreases. It appears that to estimate the effective plastic tangent modulus H^{eff} under variation of effective Young's modulus E^{eff} which have the same physical unit, MPa, and constant volume fraction give a nonlinear analytical formula leading to the loss of the physical unit. Unlike, the effective plastic tangent modulus H^{eff} obtained as a

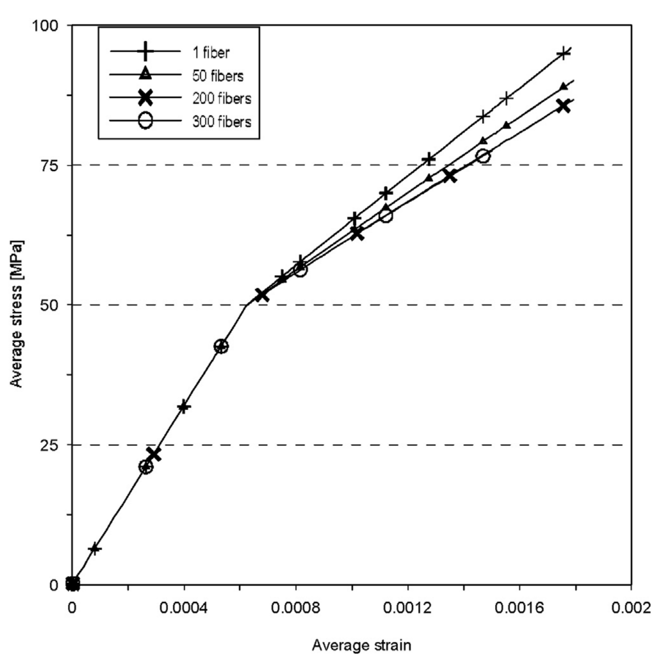


Fig. 3. Response of the transversal plane of the composite, effect of the number of fibers on the second part of the curve: the effective plastic tangent modulus as function of the number of fibers, $P = 30\% = 0.3$.

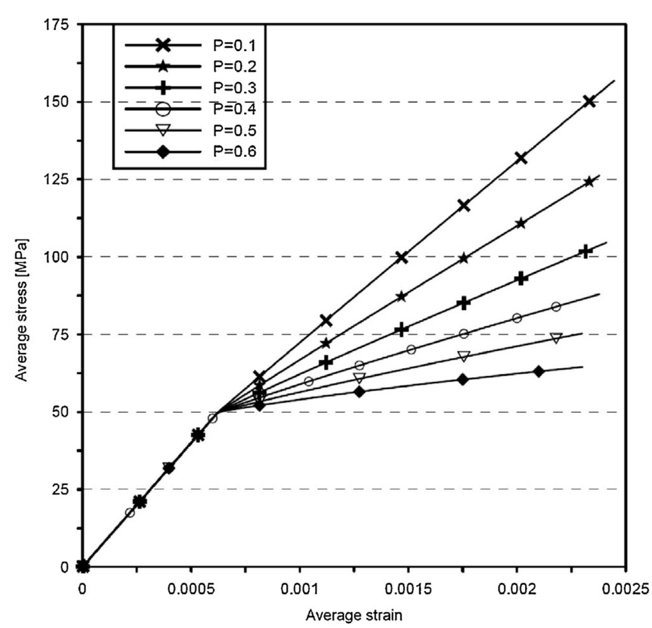


Fig. 5. Response of the composite under monotone uniaxial tension in the transversal plane of the composite, influence of volume fraction on the second part of the curve. The effective plastic tangent modulus as function of the volume fraction P.

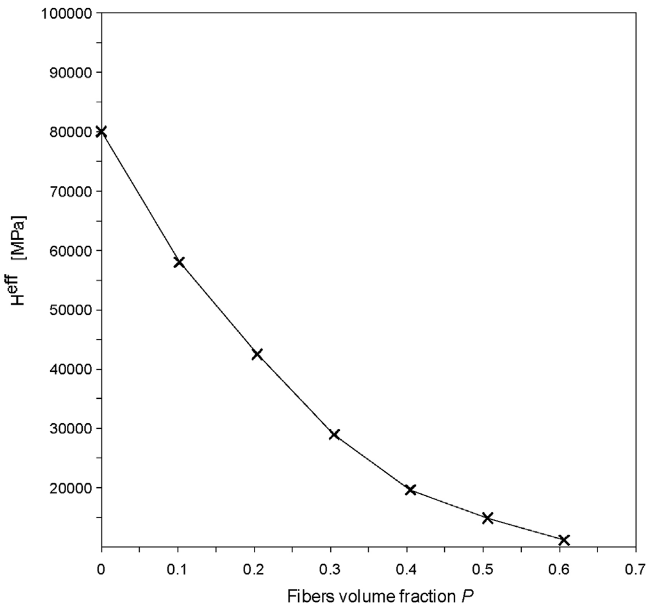


Fig. 6. Effective plastic tangent modulus in the intervals volume fractions P from 0 to 0.6, numerical results.

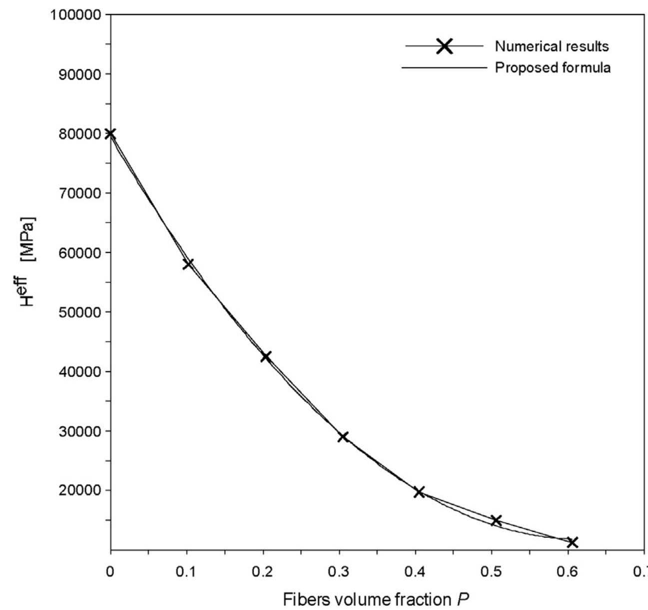


Fig. 7. Comparison between calculated and numerically evaluated effective plastic tangent modulus in the composite.

Table 1
Effective plastic tangent modulus obtained with analytical formula compared to the numerical simulations values on several volume fractions.

P	0.6	0.5	0.4	0.3	0.2	0.1
Relative error in %	5	5	2	1	1	1

function of volume fraction and Young's modulus of the two-phases homogenized, has the same unit of effective Young's modulus, MPa, when relating this one only to the volume fraction P. The results of the effective plastic tangent modulus H^{eff} in the stress–strain curves on the intervals volume fractions studied are summarized in Fig. 6.

To calculate H^{eff}, an analytical formula summarizing the relationships among the two variables: Young's modulus E_m and volume fraction P is proposed. Therefore, a simple second degree polynomial equation is chosen to fit the curve gather the effective plastic tangent modulus H^{eff} as a function of the volume fraction and effective Young's modulus. The proposed formula is:

$$H^{eff} = E_m(A_1P^2 + A_2P + A_3) \tag{1}$$

The effective Young's modulus E^{eff} is equal to E_m, where A_1, A_2 and A_3 are the three constants given by fitting with numerical results as: $A_1 \approx 2.2$; $A_2 \approx -2.74$ and $A_3 = 1$.

It should be noted that, by using the formula (1), it can easily recover the theoretical case $P = 0$, but not the case $P = 1$ because the range of variation of P is from 0 to 0.6, representing the percolation threshold beyond which the arrangement of the fibers becomes uniform. It should be noted also, that in formula (1), the value $A_3 = 1$ is very remarkable.

The values of H^{eff} simulated in preceding section and those calculated from the analytical formula obtained by (1) for different volume fractions, are reported and compared in Fig. 7. A very similar response can be observed.

The relative error in % between H^{eff}, numerically simulated, and calculated from (1) are presented in Table 1, for several volume fractions P.

Fig. 8 illustrates the tensile stress–strain curves representing the parameters of effective behavior coming from the numerical simulation and from analytical formula. The average response are checked in three volume fractions $P = 0.1$; 0.2 and 0.4. It appears that for these cases, the two curves present a very good agreement.

Comparison of the expression with numerical simulation presents a good agreement and an optimization method for computation and estimating the effective behavior of the nonlinear composite materials with randomly cylindrical distribution fibers in matrix phase.

4.3. Effect of the thickness

To study the effect of the thickness of the composite on the transverse plane (xy) behavior, a finite element computations of three-dimensional RVE have been performed to estimate the

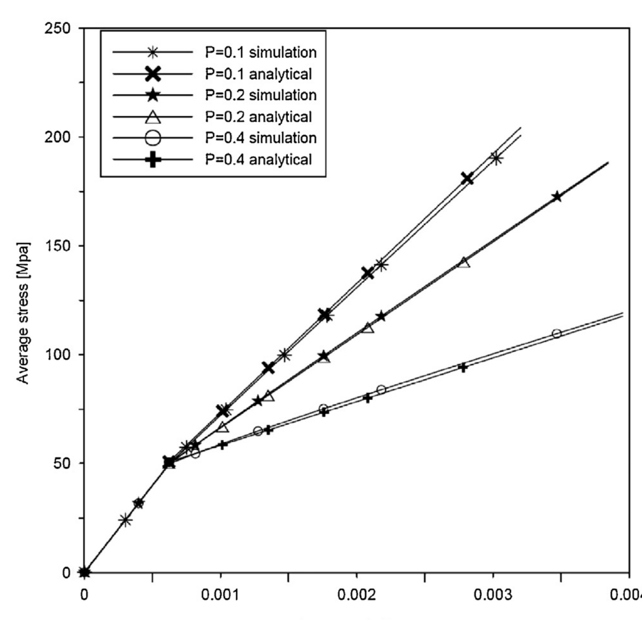


Fig. 8. Average stress–strain curves under monotone uniaxial tension: comparison between calculated and numerically evaluated tangent modulus with three fibers volume fractions.

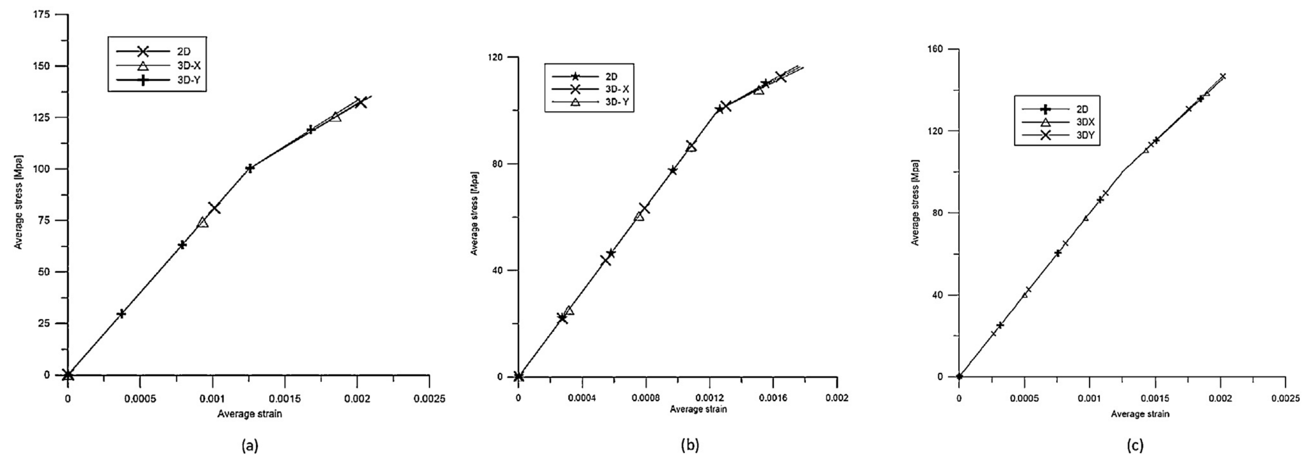


Fig. 9. Average stress–strain curves under uniaxial tension: comparison between 2D and 3D in X, Y directions: composites with $P = 0.1$; 0.2 and 0.3, respectively.

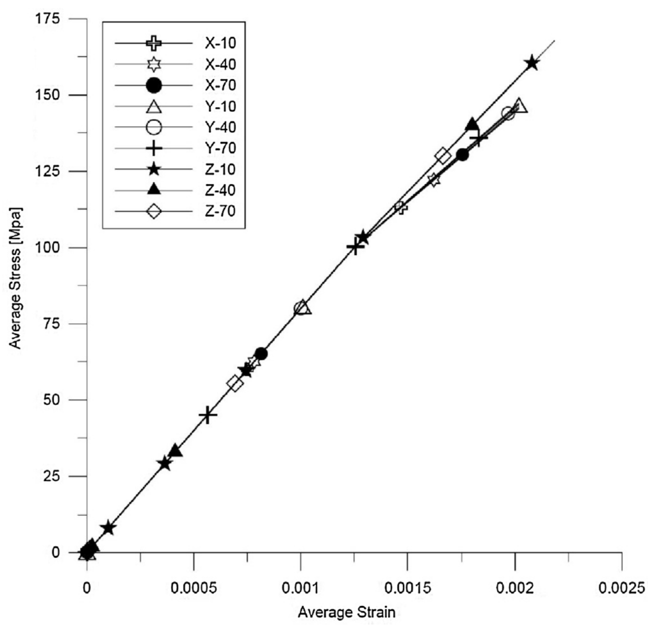


Fig. 10. Comparison of the macroscopic stress–strain response of the three-dimensional composite: influence of the thickness on the mechanical behavior, with three deferent geometries thickness $Z = 10$, 40 and 70. $X - 10$ means X tensile with thickness equal to 10.

effective elastic plastic response under tensile of the material containing different fibers volume fraction ranged from 0.1 to 0.3. The approach is validated by comparison the stress–strain curve for the 2D and 3D microstructure under the tension test.

4.3.1. *Generation of 3D microstructural finite element models*

Finite element mesh, with multiphase elements, is used to generating 3D geometry. The 3D microstructures were constructed from 2D images by means of a serial sectioning process. These images have been assembled to generate the wanted 3D RVE. The obtained microstructure consists in randomly and non-overlapping identical cylinder fibers. The mesh size used was fine enough to represent accurately the geometry of the constituents and to assure the effective response convergence. A FE mesh was then superimposed on the 3D image using quadratic brick elements. Different types of microstructures are obtained according to the volume fraction.

4.3.2. *Comparison of 2D and 3D finite element modeling results*

In this section, the mechanical properties and microstructures of the composite with three volume fractions are checked $P = 0.1$; 0.2 and 0.3 in 3D microstructural finite element models, under three tensile tests simulations independently for loading in the X, Y and Z directions, where the X and Y axes represent the 2D image coordinates. The thickness is presented along the Z axis.

The numerical computational results of tensile stress–strain curves are shown in Fig. 9(a)–(c), respectively, which the results were practically superposed in the elastic region and at the onset of plastic deformation.

4.3.3. *Influence of the thickness on the mechanical behavior*

In order to further examine the accuracy of the presented work, it's important to investigate and check the influence of the thickness on the obtained mechanical behavior. For this purpose, three different microstructures are successively considered with three different geometries thickness $Z = 10$, 40 and 70. The composite has

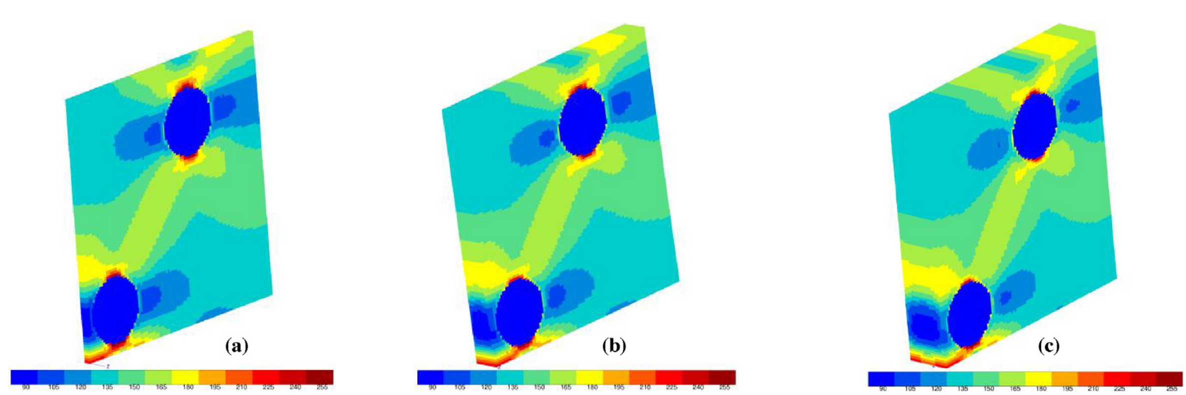
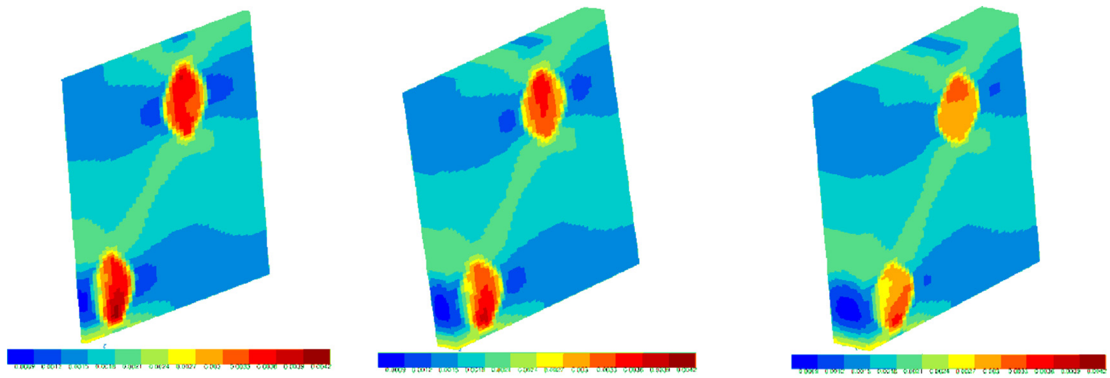


Fig. 11. Response of the 3D composite, von Mises stress distribution with 3 different geometries thickness: $Z = 10$; (b) $Z = 40$ and (c) $Z = 70$.



(a) (b) (c)

Fig. 12. Response of the 3D composite: strain along X axis, with three different geometries thickness: (a) $Z = 10$; (b) $Z = 40$ and (c) $Z = 70$.

been investigated under tensile tests simulations in the X, Y and Z directions. The results was checked by comparing the behavior presented in the stress–strain curve plotted in Fig. 10.

Fig. 10 shows that the tensile strain stress curve on the X and Y directions are initially linear and lie on top of each other, and then it's a whole different by comparing to the Z direction results in the second portion of the curves after the yield point. Examples of the evolution of stress and strain during uniaxial tensile test independently in the X direction are plotted in Figs. 11 and 12 respectively for the three different thicknesses studied.

5. Discussion and conclusion

Homogenization methods are powerful tools to simulate the mechanical behavior of heterogeneous materials, and in particular of composites, at a very reasonable computational cost. Better approximations have been developed over the years, which allow to take into account the effect of many parameters such as the volume fraction of the phases in the composite. The extension of these methods to the plastic regime in which the strain localization during plastic deformation is more difficult to estimate.

This paper deals with some major improvements of the formulation constitutive laws in order to compute the macroscopic behavior of two-phase elasto-plastic composites. The effective mechanical behavior of elastic matrix reinforced with a random distribution of an elastic-perfectly plastic identical parallel cylindrical fibers was obtained by the finite element simulation of the microstructure and by homogenization methods.

The parameters of effective behavior of the composite are expressed. They are the effective elastic tangent modulus E^{eff}, the effective plastic tangent modulus H^{eff} and the effective yield stress R_0^{eff}.

Having considered, a same elastic modulus in the two phases of the composite, in the elastic zone the global composite behavior is identical to the behavior of the two phases in terms of elastic tangent modulus. When keeping constant the volume fraction and Young's modulus, and varying the yield stress, it found that the average stress–strain curve gives in formation only about effective yield stress of the composite R_0^{eff} and has no effect on the effective tangent modulus H^{eff}. The R_0^{eff} is equal to the perfectly plastic fibers R_0^f.

The third important parameter of the behavior with linear hardening is the effective tangent modulus H^{eff} of the composite investigated. As a consequence of precedent remark, the potential will given as a function of two parameters: Young modulus and volume fraction.

To estimate the effective plastic tangent modulus H^{eff} under variation of effective Young's modulus E^{eff} which have the same unit MPa, and constant volume fraction give a nonlinear analytical formula leading to the loss of the physical unit. Unlike, the effective plastic tangent modulus H^{eff} obtained as a function of volume fraction and young modulus of the two-phases homogenized, has the same unit of effective Young's modulus MPa when relating this one only to the volume fraction P. An analytical formula presented in Eq. (2) summarize the relationships among the two variables: the volume fraction and effective Young's modulus.

In conclusion, the need of rapid and efficient methods of estimating the nonlinear effective behavior of composite materials has long been recognized. The present work allowed to construct an analytical expression enables to predict the effective mechanical behavior of elastic matrix reinforced with a random distribution of an elastic-perfectly plastic identical parallel cylindrical fibers using the volume fraction and effective young modulus of each component. Comparing the analytical expression with numerical simulation gives a very good agreement, with a maximum relative error not exceeding 5%.

The numerical simulation by finite elements of a three-dimensional representative volume element of the composite microstructure is used to investigate the influence of the thickness on the mechanical behavior obtained in 2D results. The results with versions thickness obtained were practically the same as those given in 2D results. The quality of the results found demonstrated that the 2D RVE was large enough to provide a response of the macroscopic behavior of composite.

References

Berveiller, M., Zaoui, A., 1979. An extension of the self-consistent scheme to plastically owing polycrystals. J. Mech. Phys. Solids 26, 325–344.

Besson, J., Cailletaud, G., Chaboche, J.L., Forest, S., 2001. Mécanique non linéaire des matériaux. Hermès Science.

Bornert, M., Bretheau, T., Gilormini, P., 2001. Homogénéisation en mécanique des matériaux. Hermès Science.

Camacho, C.W., Tucker, C.L., Yalvac, S., McGee, R.L., 1990. Stiffness and thermal expansion predictions for hybrid short fiber composites. Polym. Compos. 11 (4), 229–239.

Chaboche, J.L., Kanoute, P., Roos, A., 2005. On the capabilities of mean-field approaches for the description of plasticity in metal matrix composites. Int. J. Plast. 21 (7), 1409–1434.

Christensen, R.M., 1979. Mechanics of Composite Materials. Wiley, New York, pp. 84–89.

Dasgupta, A., Bhandarkar, S.M., 1992. A generalized self consistent Mori–Tanaka scheme for fiber-composites with multiple interphases. Mech. Mater. 14, 67–82.

Gilormini, P., 1995. Insuffisance de l'extension classique du modèle auto-cohérent au comportement non linéaire. C. R. Acad. Sci. Paris Ser. IIb 320, 115–122.

Guinovart-Díaz, R., Bravo-Castillero, J., Rodríguez-Ramos, R., Sabina, F.J., 2001. Closed-form expressions for the effective coefficients of fiber-reinforced

composite with transversely isotropic constituents I. Elastic and hexagonal symmetry. J. Mech. Phys. Solids 49, 1445–1462.
Guinovart-Díaz, R., López-Realpozo, J.C., Rodríguez-Ramos, R., Bravo-Castillero, J., Ramírez, M., Camacho-Montes, H., 2011. Influence of parallelogram cells in the axial behavior of fibrous composite. Int. J. Eng. Sci. 49, 75–84.
Guinovart-Díaz, R., Rodríguez-Ramos, Bravo-Castillero, J., López-Realpozo, J.C., Sabina, F.J., Sevostianov, I., 2013. Effective elastic properties of a periodic fiber reinforced composite with parallelogram-like arrangement of fibers and imperfect contact between matrix and fibers. Int. J. Eng. Sci. 58, 2–10.
Hashin, Z., 1979. Analysis of properties of fiber composites with anisotropic constituents. J. Appl. Mech. 46, 543–550.
Hill, R., 1965. A self-consistent mechanics of composite materials. J. Mech. Phys. Solids 13 (4), 213–222.
Hui-Zu, S., Tsu-Wei, C., 1995. Transverse elastic moduli of unidirectional fiber composites with fiber/matrix interfacial debonding. Combust. Sci. Technol. 53, 383–391.
Hutchinson, J.W., 1970. Elastic–plastic behavior of polycrystalline metals and composites. Proc. Roy. Soc. London A 355, 101–127.
Jeulin, D., Ostoja-Starzewski, M., 2001. Mechanics of Random and Multiscale Microstructures CISM Lecture Notes. 430. Springer-Verlag.
Ju, J.W., Sun, L.Z., 2001. Effective elastoplastic behavior of metal-matrix composites containing randomly located aligned spheroidal inhomogeneities Part I: micromechanics-based formulation. Int. J. Solids Struct. 38, 183–201.
Kanit, T., Forest, S., Galliet, I., Mounoury, V., Jeulin, D., 2003. Determination of the size of the representative volume element for random composites: statistical and numerical approach. Int. J. Solids Struct. 40, 3647–3679.
Lielens, G., (PhD thesis) 1999. Micro–macro modeling of structured materials. Université Catholique de Louvain, Belgium.
Molkov, B.A., Pobedria, B.E., 1985. Effective characteristic of fibrous unidirectional composite with periodic structure. Mech. Solids 2, 119–129.
Moravec, F., Roman, S., 2009. Numerical computing of elastic homogenized coefficients for periodic fibrous tissue. Appl. Comput. Mech. 3, 141–152.
Mori, T., Tanaka, K., 1973. Average stress in the matrix and average elastic energy of materials with misfitting inclusions. Acta Metall. 21, 571–574.
Nemat-Nasser, S., Hori, M., 1993. Micromechanics: Overall Properties of Heterogeneous Materials. North-Holland, London.
Nemat-Nasser, S., Hori, M., 1999. Micromechanics: Overall Properties of Heterogeneous Materials, second ed. Elsevier, Amsterdam.
Pettermann, H., Plankesteiner, A.F., Bohm, H.B., Rammerstorfer, F.G., 1999. A thermo-elasto-plastic constitutive law for inhomogeneous materials based on an incremental Mori–Tanaka approach. Compos. Struct. 71, 197–214.
Pierard, O., 2004. Micro/macro study of reinforced polymers and polymeric composites. Mathematical modeling and numerical simulation of elasto-viscoplastic behavior and thermal coupling. DEA graduation report. Université Catholique de Louvain, Belgium.
Ponte-Castañeda, P., Suquet, P., 1987. Nonlinear composites. Adv. Appl. Mech. 34.
Ponte-Castañeda, P., Suquet, P., 1998. Nonlinear composites. Adv. Appl. Mech. 34, 171–302.
Raimondo, L., Iannucci, L., Robinson, P., Curtis, P.T., 2012. Modeling of strain rate effects on matrix dominated elastic and failure properties of unidirectional fiber-reinforced polymer–matrix composites. Comput. Sci. Technol. 72, 819–827.
Rodríguez-Ramos, R., Sabina, F.J., Guinovart-Díaz, R., Bravo-Castillero, J., 2001. Closed-form expressions for the effective coefficients of a fiber-reinforced composite with transversely isotropic constituents I. Elastic and square symmetry. Mech. Mater. 33, 223–235.
Rodríguez-Ramos, R., Berger, H., Guinovart-Díaz, R., López-Realpozo, J.C., Würkner, M., Gabbert, U., Bravo-Castillero, J., 2012. Two approaches for the evaluation of the effective properties of elastic composite with parallelogram periodic cells. Int. J. Eng. Sci. 58, 2–10.
Sanchez-Palencia, E., Zaoui, A., 1987. Homogenization Techniques for Composite Media. Lecture Notes in Physics. 272. Springer-Verlag, Berlin.
Suquet, P., 1997. Continuum Micromechanics. CISM Courses and Lectures No. 377. Springer-Verlag, Berlin.
Swan, C.C., Kosaka, I., 1997. Voigt-Reuss topology optimization for structures with linear elastic material behaviors. Int. J. Numer. Meth. Eng. 40, 3033–3057.
Tandon, G.P., Weng, G.J., 1988. A theory of particle-reinforced plasticity. J. Appl. Mech. 55, 126–135.
Tuncer, E., 2005. Numerical calculations of effective elastic properties of two cellular structures. J. Phys. D: Appl. Phys. 38, 497–503.
Willis, J., 1981. Variational and related methods for the overall properties of composites. Adv. Appl. Mech. 21, 1–78.