

MICROMECHANICAL CHARACTERIZATION OF THE EFFECTIVE PROPERTIES FOR ANGULAR PIEZOELECTRIC FIBROUS COMPOSITES WITH IMPERFECT CONTACT CONDITION

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Abstract. A fiber-reinforced periodic piezoelectric composite where the constituents exhibit transverse isotropic properties, is considered. The fiber cross-section is rhombic and the periodicity is the same in two oblique directions. Imperfect mechanic contact conditions at the interphase between the matrix and fibers are represented in parametric form. The effective piezoelectric properties are obtained by means of the Asymptotic Homogenization Method (AHM). Some comparisons with other theoretical models are shown.

Keywords: Asymptotic homogenization, piezoelectric composites, linear spring interface model, imperfect contact

1. INTRODUCTION

The study of contact phenomena and the modelling of interfaces between two solids gain special importance in the effective property determination of composite media. In view of the growing interest on nano-scale phenomena, there has been a series of recent contributions in the literature on the effect of interfaces in solids (Miller and Shenoy, 2000, Sharma *et al.* 2003, Duan *et al.* 2005, Dingreville *et al.* 2005).

This work is motivated by the interest to study the influence of imperfect contact over the effective piezoelectric response of oblique fibrous composites. It is an extension of previous results (Bravo Castillero *et al.* 2001, Guinovart-Díaz *et al.* 2001, Sabina *et al.* 2001) where perfect contact for elastic or piezoelectric composites is considered. On the other hand, in this contribution the authors extend former researches (Molkov and Pobedria 1988, Rodríguez-Ramos *et al.* 2009) to oblique distribution of the periodic cells, where the behaviour of elastic and piezoelectric composite with hexagonal and square array under imperfect bonding contact is studied.

The applications of the method range from process modeling of ductile fracture in metals to composite materials and systems for sensing, actuation and control of micro indentation.

2. STATEMENT OF THE PROBLEM

A two-phase periodic composite is considered here which consists of rhombic array of identical parallel circular cylinders embedded in a homogeneous medium. The cylinders are infinitely long. The material electro-elastic properties of each phase belong to the crystal class δmm , where the axes of material and geometric symmetry are both parallel to the x_3 direction. The governing electro-elastic equations for this kind of materials are the Navier equations of linear elasticity and Maxwell's quasistatic equations for the mechanical displacement $\mathbf{u}=(u,v,w)$ and electric field $\mathbf{E}=(E_1,E_2,E_3)$. They become coupled equations for \mathbf{u} and \mathbf{E} through the constitutive relations of the medium. The state, which is of particular interest here, is characterized by an out-of-plane mechanical displacement w and an in-plane electric field of components E_1 and E_2 .

The main aim of this paper is the determination of the effective properties for the antiplane problems using AHM as in Sabina *et al.* (2001). In this case the relevant constitutive relations are

$$\sigma_{\lambda 3}=2C_{\lambda 3 \lambda 3} \varepsilon_{\lambda 3}-e_{\lambda 3} E_{\lambda}, D_{\lambda}=2e_{\lambda \lambda 3} \varepsilon_{\lambda 3}+d_{\lambda \lambda} E_{\lambda}, \quad (1)$$

where $\sigma_{\lambda 3}$, $C_{\lambda 3 \lambda 3}$, $\varepsilon_{\lambda 3}$, $e_{\lambda \lambda 3}$, $d_{\lambda \lambda}$ are the stress components, the longitudinal shear modulus, strain, the transverse permittivity and shear stress piezoelectric coefficient respectively; D_{λ} in-plane electric displacement components for $\lambda=1,2$. Note the differential relations $2\varepsilon_{\lambda 3}=w_{,\lambda}$, $E_{\lambda}=-\varphi_{,\lambda}$, where φ is the associated electric potential and the comma notation is understood to denote differentiation with respect to x_i i.e., $\varphi_{,i} \equiv \partial \varphi / \partial x_i$.

The equilibrium equations in the composite are

$$\sigma_{13,l} + \sigma_{23,2} = f, \quad D_{1,l} + D_{2,2} = 0, \quad (2)$$

where f is the body force (Benveniste, 1995).

The two phases are assumed to have only an imperfect elastic contact along the interface denoted by Γ . Then, the boundary conditions of the imperfect elastic bonding will correspond to the continuity of traction across the interface but there exist jumps in the displacement, i.e., in this case jumps are in the tangential direction. Moreover, continuity conditions of potential and normal component of electric displacement remain unchanged by this effect. Thus,

$$\| \sigma_{13} n_1 + \sigma_{23} n_2 \| = 0 \quad \text{on } \Gamma, \quad (3)$$

$$\sigma_{\lambda 3}^{(l)} = \sigma_{\lambda 3}^{(2)} = (K C_{313} / R) (w^{(l)} - w^{(2)}) \quad \text{on } \Gamma, \quad \lambda = 1, 2, \quad (4)$$

$$\| D \| = 0, \quad \| D_1 n_1 + D_2 n_2 \| = 0 \quad \text{on } \Gamma, \quad (5)$$

where $n = (n_1, n_2)$ is the unit normal vector to Γ , the double bar notation is used to denote the jump of the relevant function across Γ taken from the matrix to the fiber. The superscripts 1 and 2 denote the matrix and fiber respectively; R is radius of the fiber. The parameter K ($0 \leq K < \infty$) in (4) summarizes mechanical properties of the interface. It is seen that an infinite value of this parameter implies vanishing of the displacement jump and, therefore, perfect contact are described. Zero value of the parameter implies vanishing of interface tractions and, therefore, disbonding of the two adjoining media is modelled. Eq. (4) is usually called a weak interface condition; it has been originally proposed in Goland and Reissner (1944) and later studied by Benveniste and Miloh (1986), Molkov and Pobedria (1988), Mahiou and Beakou (1998), Benveniste and Miloh (2001) and Hashin (2002), etc.

3. METHOD OF SOLUTION

Let l be the distance between the centres of two neighbouring cylinders and L the diameter of the composite. Then, when $\alpha = l/L$ is a very small number, it is possible to distinguish two spatial scales, one is x , the slow variable or global variable, and the other is $y = x/\alpha$, the fast variable or local variable. The boundary value problem (1)–(5) in the composite with imperfect bonding can be solved asymptotically using the method of two scales. The functions w_0 , φ_0 , w_l , φ_l are found to satisfy certain differential equations related to the original system in a unit cell with periodic conditions. It is a well-known derivation whose details can be found elsewhere (Andrianov *et al.* 2007) and is omitted. Of a greater interest are the so-called local (or canonical) problems associated here with the correction terms w_l , φ_l to the mean variations w_0 , φ_0 since they appear in the formulae of the effective properties. There are four of such problems, which are referred as ${}_{\lambda 3}L$ and ${}_{\lambda}I$ for each value of λ ($\lambda = 1, 2$). A pre-index is used to distinguish similar constants and functions such as displacements and potentials, which appear below. Due to the linearity of the equations (1)–(5), the corrections terms w_l , φ_l can be obtained as a linear combination of some of such displacements and potentials. This, however, will not be done here, since the main objective of this paper is the characterization of the effective properties $C_{\lambda 3 \lambda 3}$, $e_{\lambda \lambda 3}$ and $d_{\lambda \lambda}$. It requires the solution of a local problem, say, ${}_{\lambda 3}L$ and ${}_{\lambda}I$. This means that is necessary to consider, viz.,

$$\bar{C}_{\lambda 3 \lambda 3} = C_{\lambda 3 \lambda 3}^{(l)} + \langle C_{\lambda 3 \lambda 3} M_{,l} + e_{\lambda \lambda 3} N_{,l} \rangle, \quad \bar{e}_{\lambda \lambda 3} = e_{\lambda \lambda 3}^{(l)} + \langle e_{\lambda \lambda 3} M_{,l} + d_{\lambda \lambda} N_{,l} \rangle, \quad \bar{d}_{\lambda \lambda} = d_{\lambda \lambda}^{(l)} - \langle e_{\lambda \lambda 3} M_{,l} - d_{\lambda \lambda} N_{,l} \rangle, \quad (6)$$

where ${}_{\lambda 3}M \equiv M$, ${}_{\lambda 3}N \equiv N$ in this section. The angular brackets in (7) define the volume average per unit length over the area, that is $\langle F \rangle = (l/|H|) \int_H F(y) dy$ where the area of H is $|H|$. The displacement M and potential N , which appear in (6), are the unique solution of the above mentioned local problem, viz.,

$$\Delta M^{(\gamma)} = 0, \quad \Delta N^{(\gamma)} = 0, \quad \text{in } S_\gamma, \quad \| N \| = 0 \quad \text{on } \Gamma, \quad (7)$$

$$\| (e_{\lambda \lambda 3} M_{,l} - d_{\lambda \lambda} N_{,l}) n_1 + (e_{\lambda \lambda 3} M_{,2} - d_{\lambda \lambda} N_{,2}) n_2 \| = - \| e_{\lambda \lambda 3} \| n_\lambda \quad \text{on } \Gamma, \quad (8)$$

$$\langle M \rangle = 0, \quad \langle N \rangle = 0, \quad (9)$$

$$(C_{\lambda 3 \lambda 3}^{(\gamma)} M_{,l}^{(\gamma)} + e_{\lambda \lambda 3}^{(\gamma)} N_{,l}^{(\gamma)}) n_l^{(\gamma)} + (C_{\lambda 3 \lambda 3}^{(\gamma)} M_{,2}^{(\gamma)} + e_{\lambda \lambda 3}^{(\gamma)} N_{,2}^{(\gamma)}) n_2^{(\gamma)} + C_{\lambda 3 \lambda 3}^{(\gamma)} n_\lambda^{(\gamma)} = \pm K C_{\lambda 3 \lambda 3}^{(l)} \| M \| R^{-l}, \quad \gamma = 1, 2. \quad (9)$$

Note that a combination of (9) is equivalent on Γ to the condition

$$\left\| \left(C_{\lambda 3 \lambda 3} M_{,I} + e_{\lambda \lambda 3} N_{,I} \right) n_I + \left(C_{\lambda 3 \lambda 3} M_{,2} + e_{\lambda \lambda 3} N_{,2} \right) n_2 \right\| = - \left\| C_{\lambda 3 \lambda 3} \right\| n_\lambda, \quad (10)$$

where Δ is the two-dimensional Laplacian. Thus $M^{(\gamma)}$ and $N^{(\gamma)}$ are sought such that they are doubly periodic harmonic functions of the complex variable $z = y_I + iy_2$ in the square unit cell $S = S_I \cup S_2$ and $S_I \cap S_2 = \emptyset$ of periods $w_I = I$ and $w_2 = e^{iu}$. When the piezoelectric coefficients e_{113} , e_{223} vanish, there is no electro-elastic coupling. Hence the equations (7)-(10) uncouple in two independent sets. Those for $M^{(\gamma)}$ and $N^{(\gamma)}$ correspond to the antiplane elastic problem $_{\lambda 3}L$ (Lopez-Lopez *et al.* 2005).

The displacement $M^{(\gamma)}$ and potential $N^{(\gamma)}$ are the unique solution of the problem $_I I$ given by

$$\Delta M^{(\gamma)} = 0, \quad \Delta N^{(\gamma)} = 0, \quad \text{in } S_\gamma, \quad \left\| N \right\| = 0 \quad \text{on } \Gamma, \\ \left\| \left(e_{\lambda \lambda 3} M_{,I} - d_{\lambda \lambda} N_{,I} \right) n_I + \left(e_{\lambda \lambda 3} M_{,2} - d_{\lambda \lambda} N_{,2} \right) n_2 \right\| = - \left\| d_{\lambda \lambda} \right\| n_\lambda \quad \text{on } \Gamma, \quad (11)$$

$$\langle M \rangle = 0, \quad \langle N \rangle = 0, \quad (12)$$

$$\left(C_{\lambda 3 \lambda 3}^{(\gamma)} M_{,I}^{(\gamma)} + e_{\lambda \lambda 3}^{(\gamma)} N_{,I}^{(\gamma)} \right) n_I + \left(C_{\lambda 3 \lambda 3}^{(\gamma)} M_{,2}^{(\gamma)} + e_{\lambda \lambda 3}^{(\gamma)} N_{,2}^{(\gamma)} \right) n_2 + C_{\lambda 3 \lambda 3}^{(\gamma)} n_\lambda^{(\gamma)} = \pm K C_{\lambda 3 \lambda 3}^{(I)} \left\| M \right\| R^{-I}, \quad \gamma = I, 2. \quad (13)$$

a combination of (13) is equivalent on Γ to the condition

$$\left\| \left(C_{\lambda 3 \lambda 3} M_{,I} + e_{\lambda \lambda 3} N_{,I} \right) n_I + \left(C_{\lambda 3 \lambda 3} M_{,2} + e_{\lambda \lambda 3} N_{,2} \right) n_2 \right\| = - \left\| e_{\lambda \lambda 3} \right\| n_\lambda. \quad (14)$$

4. SOLUTION OF THE LOCAL PROBLEMS

In this contribution, we have used the same approach that is reported in Rodríguez-Ramos *et al.* (2009) and Sabina *et al.* (2001), methods of complex potential theory are used to solve (7)-(14).

Using expansions of harmonic functions the expression (10) and (14) we obtain,

$$\begin{aligned} \bar{C}_{\lambda 3 \lambda 3} &= C_{\lambda 3 \lambda 3}^{(I)} - 2\pi R (a_I + b_I), & \bar{e}_{\lambda \lambda 3} &= e_{\lambda \lambda 3}^{(I)} - 2\pi R \left(\frac{e_{\lambda \lambda 3}^{(I)}}{C_{\lambda 3 \lambda 3}^{(I)}} a_I - \frac{d_{\lambda \lambda}^{(I)}}{e_{\lambda \lambda 3}^{(I)}} b_I \right), \\ \bar{e}_{\lambda 3 \lambda} &= e_{\lambda 3 \lambda}^{(I)} - 2\pi R \left(\frac{e_{\lambda 3 \lambda}^{(I)}}{C_{\lambda 3 \lambda 3}^{(I)}} a_I - \frac{d_{\lambda \lambda}^{(I)}}{e_{\lambda 3 \lambda}^{(I)}} b_I \right), & \bar{d}_{\lambda \lambda} &= d_{\lambda \lambda}^{(I)} + 2\pi R (a_I + b_I), \end{aligned} \quad (15)$$

in which only the residue of $M^{(I)}$ and $N^{(I)}$ contributes towards $\bar{C}_{\lambda 3 \lambda 3}$, $\bar{e}_{\lambda \lambda 3}$ and $\bar{d}_{\lambda \lambda}$. The expressions for effective coefficients are the same as was reported by Sabina *et al.* (2001), Lopez-Lopez *et al.* (2005) for piezoelectric cases with perfect contact conditions. The coefficients a_I and b_I , however, are different in both cases. Thus, expressions for a_I , b_I are now sought from the system of infinite equations

$$\left[mI + nQ + sW \right] A + \left[pI + qQ + rW \right] B = U, \quad (16)$$

where the following notation is used: identity matrix I , symmetric matrix $W(w_{kj})$ with components $w_{kj} = \eta_{kj}$, the columns matrixes $A^T = (a_I \ a_3 \ a_5 \ a_7)$, $B^T = (b_I \ b_3 \ b_5 \ b_7)$, $U^T = (2R \ 0 \ 0 \ 0)$, the superscript T denotes transpose; the material parameters $m = \alpha_3^{-1} + \beta_3^{-1}$, $n = \alpha_I \alpha_3^{-1} + \beta_I \beta_3^{-1}$, $s = \alpha_2 \alpha_3^{-1} + \beta_2 \beta_3^{-1}$, $p = \delta_I \alpha_3^{-1} + \gamma_I \beta_3^{-1}$, $q = \delta_2 \alpha_3^{-1} + \gamma_2 \beta_3^{-1}$, $r = \delta_3 \alpha_3^{-1} + \gamma_3 \beta_3^{-1}$ and the matrix $Q(q_{kj})$ with components $q_{kj} = A_{\lambda 3} \delta_{kI} \delta_{Ij}$ whereas $A_{\lambda 3} = \begin{cases} 2R^2 \operatorname{Re}(\zeta(I/2)) & \text{si } \lambda = I, \\ 2R^2 \operatorname{Im}(\zeta(\omega_2/2)) & \text{si } \lambda = 2 \end{cases}$

where $\zeta(z) = \frac{1}{z} + \sum_{mn} \left(\frac{1}{z - \beta_{mn}} + \frac{1}{\beta_{mn}} + \frac{z}{\beta_{mn}^2} \right)$, $\beta_{mn} = m + ne^{i\theta}$, $\zeta(z)$ is the Weierstrass's function for $p = I, 3, 5, \dots$. For seek of brevity, the expression of the magnitudes $\alpha_I, \alpha_2, \alpha_3, \beta_I, \beta_2, \beta_3, \delta_I, \delta_2, \delta_3, \gamma_I, \gamma_2, \gamma_3$ that depend on $_{\lambda 3} \mathcal{X}^* = C_{\lambda 3 \lambda 3}^{(2)} / C_{\lambda 3 \lambda 3}^{(I)}$, $_{\lambda 3} \mathcal{S}^* = e_{\lambda \lambda 3}^{(2)} / e_{\lambda \lambda 3}^{(I)}$, $_{\lambda 3} \mathcal{T}^* = d_{\lambda \lambda}^{(2)} / d_{\lambda \lambda}^{(I)}$ and the imperfection parameter K are not given. The values of the previous mentioned magnitudes are different for $_{\lambda 3} L$ and $_{\lambda} I$ antiplane problems with $\lambda = I, 2$.

The limit case of perfect contact condition is derived as a particular case as $K \rightarrow \infty$. In this case, the parameters a_l , b_l are the same that in Sabina *et al.* (2001) and Bravo-Castillero *et al.* (2001). The infinite system is used such that it is truncated for obtaining a $N \times N$ order system. Interesting fact is concerned that the effective properties are monotonic function of order N of the solution of the system. The numerical results converge quit good to the exact solutions when an adequate order in the solution of the system is chosen as N increase.

5. NUMERICAL RESULTS

In order to illustrate the efficiency of the present model, some numerical examples are analyzed.

a) The material parameters used in Tab.1 are: matrix PZT-5 ($C_{1313} = 21.1Gpa$, $e_{113} = 12.3C/m^2$, $k_{11} = 8.107nC^2/Nm^2$) and fiber empty (material parameters of order zero). This table indicates the comparison between present model ($K \rightarrow 0$), the model derived in Bravo-Castillero *et al.* (2009) using Asymptotic Homogenization Method (AHM) under perfect contact condition (continuity of displacements and stress) and Tab.2 given by Jiang and Cheung (2001) for ceramic porous case. Notice a good agreement between these numerical results.

Table 1. Comparison between present model, AHM reported in Bravo-Castillero *et al.* (2009) and Table 2 given by Jiang and Cheung (2001).

Vf	$\bar{C}_{1313} (Gpa)$			$\bar{e}_{113} (C/m^2)$			$\bar{d}_{11} (nC^2/Nm^2)$		
	Jiang-Cheung	Present model	AHM	Jiang-Cheung	Present model	AHM	Jiang-Cheung	Present model	AHM
0	21,1000	21,1000	21,1000	12,3000	12,3000	12.3000	8,1070	8.1070	8.1070
0.1	17,2640	17,2640	17,2640	10,0640	10,0640	10.0638	6,6359	6.6330	6.6340
0.2	14,0670	14,0670	14,0670	8,2000	8,2000	8.2002	5,4096	5.4047	5.4066
0.3	11,3620	11,3620	11,3620	6,6231	6,6231	6.6333	4,3716	4.3653	4.3681
0.5	7,0333	7,0333	7,0333	4,1000	4,1000	4.1000	2,7102	2.7023	2.7067
0.6	5,2750	5,2750	5,2750	3,0750	3,0750	3.0750	2,0350	2.0268	2.0321
0.8	2,3445	2,3444	2,3445	1,3667	1,3667	1.3667	0,9095	0.9008	0.9079
0.9	1,1105	1,1105	1,1105	0,6474	0,6474	0.6474	0,4355	0.4267	0.4347

b) In Tab.2 a comparison between the present model for perfect contact ($K = 10^5$) with different angle of fiber distribution and the results of Tab.3 for square array reported in Jiang and Cheung (2001) is given. The material parameters used are: matrix PZT-5 and polymer fiber $C_{1313} = 0.64Gpa$, $e_{113} = 0$, $k_{11} = 0.0797nC^2/Nm^2$. This table shows that this approach is suitable.

Table 2. Comparison between the present model and the results of Table 3 reported in Jiang and Cheung (2001) for PZT-5/polymer composite.

Vf	$\bar{C}_{1313} (Gpa)$			$\bar{e}_{113} (C/m^2)$			$\bar{d}_{11} (nC^2/Nm^2)$		
	Jiang-Cheung 90°	Present model		Jiang-Cheung 90°	Present model		Jiang-Cheung 90°	Present model	
		90°	50°		90°	50°		90°	50°
0	21,100	21.1000	21.1000	12,300	12.3000	12.3000	8,1070	8.1070	8.1070
0.1	17,473	17.4720	17.4380	10,064	10.0640	10.0420	6,6592	6.6592	6.6449
0.2	14,419	14.4180	14.3010	8,2007	8.2007	8.1246	5,4488	5.4488	5.4002
0.3	11,812	11.8120	11.5830	6,6238	6.6238	6.4768	4,4217	4.4217	4.3277
0.5	7,5992	7.5992	7.1095	4,1006	4.1006	3.7901	2,7731	2.7731	2.5738
0.6	5,8726	5.8726	5.2467	3,0755	3.0755	2.6806	2,1014	2.1014	1.8475
0.8	2,9754			1,3669			0,9794		
0.9	1,7482			0,6475			0,5061		

c) Numerical results using the present model for imperfect contact with different values of the imperfection parameter is listed. The composite is made of PZT-5/polymer with square cell. The used material constants are given in the previous example. Table 3 allows us to observe the effect of the imperfect contact in the composite.

Table 3. Numerical data using the present model for different values of imperfect parameter K .

Vf	$\bar{C}_{1313} (Gpa)$			$\bar{e}_{113} (C / m^2)$			$\bar{d}_{11} (nC^2 / Nm^2)$		
	$K=10^5$	$K=1$	$K=10^{-2}$	$K=10^5$	$K=1$	$K=10^{-2}$	$K=10^5$	$K=1$	$K=10^{-2}$
0	21.1000	21.1000	21.1000	12.3000	12.3000	12.3000	8.1070	8.1070	8.1070
0.1	17.2640	17.4660	17.4720	10.0640	10.0640	10.0640	6.6592	6.6592	6.6592
0.2	14.0670	14.4080	14.4180	8.2000	8.2007	8.2007	5.4488	5.4488	5.4488
0.3	11.3620	11.7990	11.8120	6.6231	6.6238	6.6238	4.4217	4.4217	4.4217
0.5	7.0335	7.5826	7.5990	4.1000	4.1006	4.1006	2.7731	2.7731	2.7731
0.6	5.2752	5.8551	5.8724	3.0750	3.0754	3.0755	2.1014	2.1014	2.1014
0.8	2.3447	2.9569	2.9752	1.3667	1.3669	1.3669	0.9795	0.9795	0.9795
0.9	1.1107	1.7295	1.7480	0.6475	0.6475	0.6475	0.5061	0.5061	0.50614

d) In order to verify the computational implementation and illustrate the utility of formulae (15)-(16) for checking numerical codes, a comparison between the present analytical model for perfect contact ($K = 10^5$) and a finite element model reported in Kar-Gupta and Venkatesh (2006) is given. The composite considered is a barium titanate 3–1 longitudinally porous piezoelectric material with square periodic distributions of empty fibers. The material properties used to calculate the results shown in Tab.4 were taken from Tab.1 of Kar-Gupta and Venkatesh (2006) (matrix $BaTiO_3$ $C_{1313} = 43.86 Gpa$, $e_{113} = 11.4 C / m^2$, $k_{11} = 12.8 nC^2 / Nm^2$ and fiber empty). A good agreement between the results for the elastic, piezoelectric and dielectric effective coefficients from the analytical and numerical models can be observed for the whole range of empty-fiber volume fractions.

Table 4. Present analytical model ($K = 10^5$) and a finite element model reported in Kar-Gupta and Venkatesh (2006).

Vf	$\bar{C}_{1313} (Gpa)$		$\bar{e}_{113} (C / m^2)$		$\bar{d}_{11} (nC^2 / Nm^2)$	
	Kar-Gupta and Venkatesh	Present model	Kar-Gupta and Venkatesh	Present model	Kar-Gupta and Venkatesh	Present model
0	43.86	43.86	11.40	11.40	12.80	12.80
0.1	35.94	35.89	9.35	9.33	10.49	10.47
0.3	23.65	23.62	6.15	6.14	6.091	6.89
0.5	14.36	14.62	3.73	3.80	4.192	4.27

6. CONCLUSIONS

In this work, an asymptotic approach for simulation of the elastic imperfect bonding is proposed. We introduce between the matrix and inclusions a set of elastic springs that transmits a load from the matrix to the inclusion proportional to the displacement jump across the “matrix-inclusion” interface. Besides, the former perfect contact models reported by the authors verify the reported experimental results.

The local problems are solved and developed solutions are valid for all values of the components volume fractions and properties. The analytic expressions of the effective properties reported in Sabina *et al.* (2001) and Lopez-Lopez *et al.* (2005) for perfect contact and the present model with square cell for imperfect contact coincide when the stiffness value of K is very high.

The debonding parameter K can be considered as a kind of phenomenological value. Unfortunately, up to now, the authors have not found experimental data related to characterization of the effective coefficients in piezoelectric composites under imperfect contact conditions.

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8. REFERENCES

- Andrianov, I.V., Bolshakov, V.I., Danishevs'kyy, V.V., Weichert, D., 2007. "Asymptotic simulation of imperfect bonding in periodic fibre-reinforced composite materials under axial shear", *International Journal of Mechanics Science*, Vol. 49, pp. 1344-1354.
- Benveniste, Y. and Miloh, T., 1986. "The effective conductivity of composites with imperfect thermal contact at constituent interfaces", *International Journal of Engineering Science*, Vol. 24, No. 9, pp. 1537-1552.
- Benveniste, Y. 1995. "Correspondance relations among equivalent classes of heterogeneous piezoelectric solids under anti-plane mechanical and in-plane electric fields", *Journal of the Mechanics and Physics of Solids*, Vol. 43, pp. 553-571.
- Benveniste, Y., Miloh, T., 2001. "Imperfect soft and stiff interfaces in two dimensional elasticity", *Mechanics of Materials*, Vol. 33, pp. 309-323.
- Bravo-Castillero J., Guinovart Díaz R, Sabina F. J., Rodríguez Ramos R., 2001. "Closed-form expressions for the effective coefficients of a fiber-reinforced composite with transversely isotropic constituents-II. Piezoelectric and square symmetry", *Mechanics of Materials*, Vol. 33, No. 4, pp. 237-248.
- Bravo-Castillero J., Guinovart Díaz R, Camacho-Montes, H., Pérez-Fernández, L.D., 2010. "Homogenization and universal relations in magneto-electro-elastic perforated structures". *Proceedings of the XI Pan-American Congress of Applied Mechanics (PACAM XI)*, January 04-08, 2010, Foz do Iguaçu, PR, Brazil.
- Dingreville, R., Qu, J., Cherkaoui, M., 2005. "Surface free energy and its effect on the elastic behavior of nanosized particles, wires and films", *Journal of the Mechanics and Physics of Solids*, Vol. 53, pp. 1829-1854.
- Duan, H.L., Wang, J., Huang, Z.P., Karihaloo, B.L., 2005. "Size-dependent effective elastic constants of solids containing nano-inhomogeneities with interface stress", *Journal of the Mechanics and Physics of Solids*, Vol. 53, pp. 1574-1596.
- Goland M., Reissner, E., 1944. "The stresses in cemented joints", *Journal of Applied Mechanics*, Vol. 11, pp. 17-27.
- Hashin, Z., 2002. "Thin interphase/imperfect interface in elasticity with application to coated fiber composites", *Journal of the Mechanics and Physics of Solids*, Vol. 50, pp. 2509-2537.
- Jiang, C.P. and Cheung, Y.K., 2001. "An exact solution for the three-phase piezoelectric cylinder model under antiplane shear and its applications to piezoelectric composites", *International Journal of Solids and Structures*, Vol. 38, pp. 4777-4796.
- Kar-Gupta R, Venkatesh T.A., 2006. "Electromechanical response of porous piezoelectric materials", *Acta Materialia*, Vol. 54, pp. 4063-4078.
- López-López, E., Sabina, F.J., Bravo-Castillero, J., Guinovart-Díaz, R., Rodríguez-Ramos, R., 2005. "Overall electromechanical properties of a binary composite with 622 symmetry constituents. Antiplane shear piezoelectric state", *International Journal of Solids and Structures*, Vol. 42, pp. 5765-5777.
- Mahiou, H., Beakou, A., 1998. "Modelling of interfacial effects on the mechanical properties of fibre-reinforced composites", *Composites Part A*, Vol. 29A, pp. 1035-1048.
- Miller, R.E., Shenoy, V.B., 2000. "Size-dependent elastic properties of nano-sized structural elements", *Nanotechnology* Vol. 11, pp. 139-147.
- Molkov, V.A., Pobedria, B.E., 1988. "Effective elastic properties of a composite with elastic contact", *Izvestia Akademii Nauk SSR, Mekhanika Tverdogo Tela*. No. 1, pp. 111-117.
- Rodríguez-Ramos, R., Guinovart-Díaz, R., López-Realpozo, J.C., Bravo-Castillero, J., Sabina, F.J., 2009. "Influence of imperfect elastic contact condition on the antiplane effective properties of piezoelectric fibrous composites", *Archive of Applied Mechanics*, 10.1007/s00419-009-0320-3
- Sabina F.J, Rodríguez Ramos R., Bravo Castillero J., Guinovart Díaz R., 2001. "Closed-form expressions for the effective coefficients of fibre-reinforced composite with transversely isotropic constituents-II: Piezoelectric and hexagonal symmetry", *Journal of the Mechanics and Physics of Solids*, Vol. 49, pp. 1463-1479.
- Sharma, P., Ganti, S., Bhate, N., 2003. "Effect of surfaces on the size-dependent elastic state of nanoinhomogeneities", *Applied Physics Letters*, Vol. 82, pp. 535-537.

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