Elastic behavior of composites with imperfect interphase

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Abstract

In this contribution, the effective elastic moduli are obtained, by means of the Asymptotic Homogenization Method (AHM), for two different types of two-phase periodic composites: fibrous and laminate composites with imperfect contact condition (*spring type*). In both cases the constituents exhibit transversely isotropic properties. This work is an extension of the results before reported, where only perfect contacts for elastic or piezoelectric composites were considered. Some numerical examples and comparisons with other theoretical results demonstrate that the present model by AHM is efficient for the analysis of composites with presence of imperfect interphase.

Introduction

In most composites, the fiber-matrix adhesion is imperfect; the continuity conditions for stresses and displacements are not satisfied. Thus various approaches have been used, in which the bond between the reinforcements and the matrix is modeled by an interphase with specified thickness. The most common considered assumptions, it is to suppose that the contrast or jump of the displacements in the interface are proportional to the corresponding component of the tension in the interface in terms of a parameter given by the constant of a spring. This type of imperfect contact (*spring type*) in the interphases of the composites was initially investigated in [1] and has been later on used, for instance, in [2-4]. In this work, analytic expressions for the effective coefficients of two types of composites: fibrous and laminate, with anisotropic elastic constituents under *spring type* of imperfect contact condition are calculated using the AHM [5-7].

Constitutive equations

Consider elastic materials that respond linearly to changes in the mechanical stress and strain tensors. These assumptions are compatible with the polymers and composites in current use. The behavior of the elastic medium is described, for each phase, by the $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$ constitutive equations relate the stress σ_{ij}, strain ε_{kl}, where C_{ijkl} is a piecewise-constant fourth-order elasticity tensor. Imperfect contact condition (*spring type*) at the interface Γ can be written as follows $\sigma_{ij} = \pm \mathrm{K}(u_1 - u_2)$, where K denotes the imperfect parameter at the interface (for example, K can be considered as a frictional coefficient on the contact region Γ between the matrix and inclusions). The sub indices 1 and 2 denote the matrix and inclusions respectively.

Asymptotic Homogenization Method (AHM)

The constitutive relations of the linear elasticity theory for a heterogeneous and periodic medium X, is characterized by the Y-periodic function C_{ijkl}. Y denotes the unit periodic cell with length $l \ll 1$. By mean of AHM, the constitutive relations with rapidly oscillating material coefficients are transformed in new physical relations with constant coefficients $\overline{C}_{ijkl}$, which represent the elastic properties of an equivalent homogeneous medium and are called the effective coefficients of X.

Application of AHM to laminate composite

Let us consider now an elastic laminate composite, made of anisotropic homogeneous layers perpendicular to the x_3-axis. The Local Problem, see for instance [7], consists in to find the 1-periodic functions $N_{mpq}(y)$, where $y \equiv y_3$ it is the fast variable, such that $\mathrm{D}\left(C_{i3pq} + C_{i3m3}\,\mathrm{D}\,N_{mpq}(y)\right) = 0$, being $\mathrm{D} \equiv d/dy$. The solution of such problem on the periodic cell satisfies the boundary condition $u(0) = 0$, the periodic condition $u(0) = u(1)$ and the mechanic imperfect contact conditions $\sigma_{i3} = \pm \mathrm{K}(u_3^{(1)} - u_3^{(2)})$ (i=1,2). Since we are dealing with a laminate composite, in general, only the coefficients in the tangential direction to the distribution of the layers, that is, $\overline{C}_{1313}$ and $\overline{C}_{2323}$, are different from those effective elastic coefficients with perfect contact condition in [7]. For a composite with transversely isotropic constituents the analytic expression of effective axial shear is:

$$\overline{p} \equiv \overline{C}_{1313} = \overline{C}_{2323} = \left(\left\langle p^{-1}\right\rangle + (lK)^{-1}\right)^{-1} = \left(\left\langle p^{-1}\right\rangle + (lK)^{-1}\right)^{-1}. \tag{1}$$

Application of AHM to elastic fibrous composite

In the two phase fibrous composite each fiber is a long continuous circular cylinder of radius R; its axis of symmetry is parallel to the x_3-axis. Moreover, the fibers are arranged periodically in a hexagonal or square array (see Fig. 1(b)). For sake of brevity, the solution of only one local problem is reported, in particular the elastic anti-plane problem ${}_{13}L$. Therefore, only one effective coefficient $\overline{p}$ is derived. In fact, the equations over the periodic cell are $p_\alpha u_{3,kk}^{(\alpha)} = 0$, ($\alpha = 1{,}2$ denotes the constituents of the composite) with imperfect contact written in the form $\left(\sigma_{3j}^{(\alpha)} n_j^{(\alpha)} + p_\alpha n_1^{(\alpha)}\right) = \mp K p_1 (u_3^{(1)} - u_3^{(2)})/R$, on Γ.

The methods of a complex variable z are used here. Following the same procedure that in [5] but using the imperfect contact condition we obtain the same analytical expressions that perfect contact reported in [5-6],

$$\overline{p} = p_1(1 - 2\pi R a_1). \tag{2}$$

The unknown constant $-a_1$ can be found upon the solution [8] of the following infinite system

$$a_p - \beta_1 A\delta_{1p} - \beta_2 \sum_{k=1}^{\infty}{}^{*} a_k \eta_{kp} = \beta_1 R\delta_{1p} \text{ where } \beta_p = \frac{\chi^* K - p\chi^* - K}{\chi^* K + p\chi^* + K} \text{ and } \chi^* = \frac{p_2}{p_1}.$$

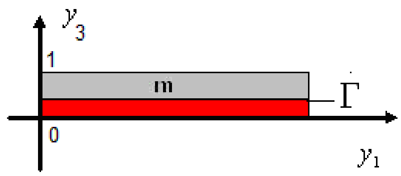

Fig.1 (a) Periodic laminate cell.

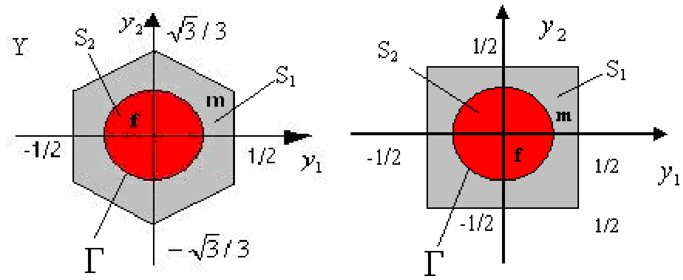

Fig.1(b) Hexagonal and square periodic cell.

Analysis of the results

1) Formula (1) is reduced to isotropic case. On the other hand, it is also possible to verify in a simple way that the effective coefficients obtained in (1) reproduce the limit case of the effective coefficients for perfect contact. It is achieved when the spring parameter K approaches to infinite. Fig.2a shows the behaviour of the elastic shear modulus $\bar{p}$ for a two-layered composite of $BaTiO_3$ / PZT-7A for different values of imperfect parameter. The material constants used in the calculations were taken from [9]. Fig. 2a also shows the modulus $\bar{p}$ for perfect adhesion between the layers ($K = \infty$). The influence of the imperfect contact for different values of the parameter K is observed. Notice that all curves follow the same trend. The hardening of $\bar{p}$ arises for a fix value of the volume fraction ($BaTiO_3$) when increases the parameter K. The same remarks can be pointed out when the volume fraction ($BaTiO_3$) increases for a fix value of K.

2) Fig.2b shows normalized effective shear coefficient (2) for $BaTiO_3$ / PZT-7A fibrous composite and different values of K, versus the volume fraction of $BaTiO_3$. The present solution is compared with the three-phase exact model in [10], that introduce an artificial thin bound layer (denoted by I) between the matrix and fiber with volume fraction $\eta_I = 10^{-7}$ and $p_I / p_1 = K\eta_I$ as in [3]. Exact agreement between both approaches can be observed in all range of fiber volume fraction. -It is seen that there is a significant effect of imperfection for $K \leq 10$ causing a weakening of the effective shear property. Perfect interface is achieved for $K \to \infty$ and disbond can be observed for $K \to 0$.

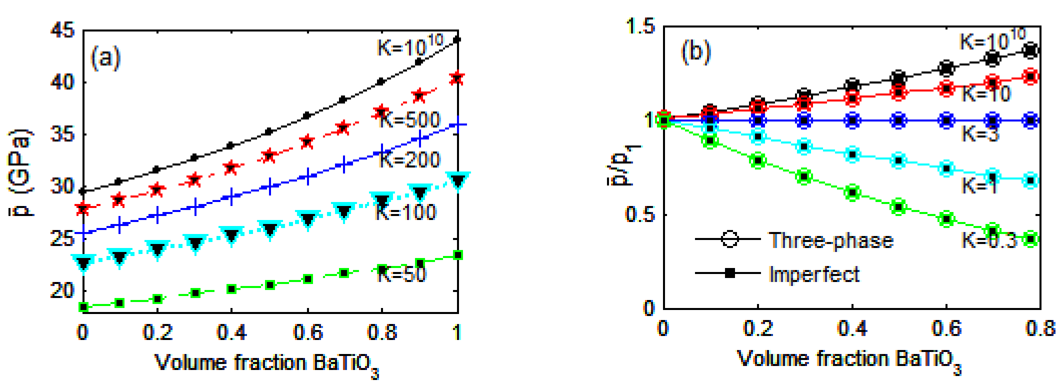

Fig.2 Behavior of the elastic shear modulus for $BaTiO_3$ / PZT-7A composite versus volume fraction of $BaTiO_3$ and different values of the imperfection parameter K. a) laminate composite. b) Comparison between three-phase and imperfect models for fibrous composite.

Conclusions

An asymptotic approach for simulation of the elastic imperfect bonding in composite materials is proposed. We introduce in the contact conditions between the matrix and inclusions a set of elastic springs that transmits a load from the matrix to the inclusion; the transmission stress is proportional to the displacement jump across the "matrix-inclusion" interface. In the asymptotic limit, we can simulate different degrees of the interfaces response. The debonding parameter K can be considered as a kind of phenomenological value.

Acknowledgements

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